Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

From (3) we have $x = \dfrac{a}{r_1{}^2 \cot \frac{1}{2}B + 2r_1 r_2 + r_2{}^2 \cot \frac{1}{2}C}$

$$= \frac{a \sin \frac{1}{2}B \sin \frac{1}{2}C}{r_1{}^2 \cos \frac{1}{2}B \sin \frac{1}{2}C + 2r_1 r_2 \sin \frac{1}{2}B \sin \frac{1}{2}C + r_2{}^2 \sin \frac{1}{2}B \cos \frac{1}{2}C}$$

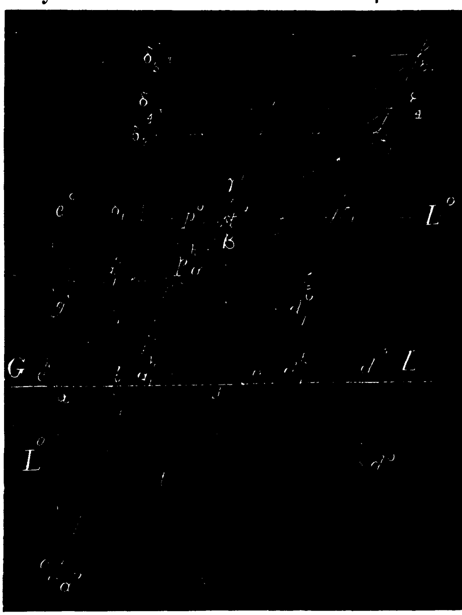
$$= \frac{r \cos \frac{1}{4}A \cos \frac{1}{4}(\pi - B) \cos \frac{1}{4}(\pi - C)}{2 \cos \frac{1}{4}\pi \cos \frac{1}{4}B \cos \frac{1}{4}C \cos \frac{1}{4}(\pi - A)} = \frac{\frac{1}{2}r(1 + \tan \frac{1}{4}B)(1 + \tan \frac{1}{4}C)}{1 + \tan \frac{1}{4}A}.$$

$$\therefore\ r_1{}^2 x = \frac{\frac{1}{2}r(1 + \tan \frac{1}{4}A)(1 + \tan \frac{1}{4}C)}{1 + \text{tsn} \frac{1}{4}B},\ r_2{}^2 x = \frac{\frac{1}{2}r(1 + \tan \frac{1}{4}A)(1 + \tan \frac{1}{4}B)}{1 + \tan \frac{1}{4}C}.$$

SOLUTION OF MR. CHURCH'S PROBLEM.

BY PROF. E. W. HYDE, ITHACA, N. Y.

"Given four points [no one point lying within the triangle formed by the other three] to construct geometrically the axis and focus of the parabola passing through them.



Solution.—Let the four points be a^0, b, c, d^0. Regard them as lying in the horizontal plane of projection, and draw a ground line GL through two of them as b and c. Take some point p [marked $p^h p^v$] in the second angle, and draw the right lines pd and pa piercing the vertical plane in d_1 and a_1. b and c are in GL and therefore in both the horizontal and vertical planes of projection.

Draw in the vertical plane a horizontal line L^0 through p^v. Now if an ellipse be drawn passing through the four points $a^0{}_1$, b, c, and $d^0{}_1$, and also tangent to the line L^0, and if p be taken as the vertex of the projecting cone, the ellipse $a^0{}_1$ b c $d^0{}_1$ t^0 will be projected into the required parabola, upon the horizontal plane. For by the construction b and c are their own projections, $a^0{}_1$ is projected into a^0, and $d^0{}_1$ into d^0, and t^0 is projected to infinity. It follows therefore that p^h t^h is a diameter of the parabola. It is not necessary to construct the ellipse. Draw by Pascal's theorem a tangent to the ellipse at

either of the given points as b. In the figure $b\,e^0$ is the tangent. Project it upon the horizontal plane into $L^0{}_1$, the tangent to the parabola at b. Having now the tangent and diameter at b, other tangents as those at c and c_1 can be drawn at once, and the curve can be constructed, and its axis and focus found.

It only remains to show how to find the point t^0 at which the ellipse through $a^0{}_1$, b, c, $d^0{}_1$ is tangent to L^0.

Suppose t^0 to have been found, and consider $a^0{}_1\,b\,c\,d^0{}_1\,t^0$ as an inscribed hexagon with the side at t^0 reduced to a point; then by Pascal's theorem the intersections of the three pairs of lines $\overline{d^0{}_1\,t^0}$ and $\overline{b\,a^0{}_1}$, $\overline{a^0{}_1\,t^0}$ and $\overline{c\,d^0{}_1}$, $\overline{b\,c}$ and L^0 must lie in the same straight line. But as the last two are parallel, the line through the three intersections will de parallel to $\overline{b\,c}$, or GL.

If therefore we draw a series of lines $\overline{\delta_2\varepsilon_2}$, $\overline{\delta_3\varepsilon_3}$ etc. parallel to GL, and draw also the lines $\overline{d^0{}_1\delta_1}$, $\overline{d^0{}_1\delta_2}$ etc. and $\overline{a^0{}_1\varepsilon_1}$, $\overline{a^0{}_1\varepsilon_2}$ etc., then the locus ot the intersections α, β, γ, etc. of these last lines will cut L^0 at the required point of contact t^0.

This locus is a conic section with its center at k the middle point of $a^0{}_1\,d^0{}_1$ which is a diameter. It has a pair of conjugate diameters parallel to $c\,d^0{}_1$ and $b\,a^0{}_1$, and its equation referred to these diameters is

$$\frac{x^2}{\frac{1}{4}a(a \pm a_1)} + \frac{y^2}{\frac{1}{4}b(b \pm b_1)} = 1;$$

in which a is the distance from the intersetion of $c\,d^0{}_1$ and $b\,a^0{}_1$ to the point $d^0{}_1$; a_1 that from the same intersection to ζ; b that from the intersection to $a^0{}_1$; and b_1 that from the intersection to η; the lines $a^0{}_1\,\zeta$ and $\eta\,d^0{}_1$ being parallel to GL.

If the upper signs of a_1 and b_1 be taken the curve is an ellipse, if the lower, it is an hyperbola. The first corresponds to the case where the lines $\delta_1\,\varepsilon_1$, $\delta_2\,\varepsilon_2$ etc. are drawn across the other angle between the lines $\overline{cd^0{}_1}$ and $\overline{ba^0{}_1}$, i. e. so as to cut one of them on the same side of their intersection on which $a^0{}_1$ and $d^0{}_1$ are situated, and the other on the other side, while the second is the case in the figure. If $a^0{}_1$ and $d^0{}_1$ are equally distant from GL, or if $\overline{ba^0{}_1}$ and $\overline{cd^0{}_1}$ are parallel the locus reduces to two intersecting right lines.

QUADRATURE OF THE CIRCLE.

BY PROF. AUGUST ZIELINSKI, AUGUSTA, GEORGIA.

The Geometers of Antiquity used to distinguish, in the solution of mathematical problems, the geometrical solutions and the mechanical ones—the